mass, we have for the elementary moment about the axis of the plate

$$dM = \delta\Omega^2 \frac{b}{a}(a^2 - x^2)^{\frac{1}{2}}\, xzdxdz,$$

where z is the distance from the rotation axis to the minor axis of the elliptic lamina. Hence if d be the distance from the fixed axis to the centre line of the plate, we have for the total bending moment

$$M = 2\delta\Omega^2 \frac{b}{a} \int_{d-\frac{1}{2}c}^{d+\frac{1}{2}c} \int_{0}^{a} (a^2 - x^2)^{\frac{1}{2}}\, xzdxdz$$

$$= \tfrac{2}{3}\delta\Omega^2 ba^2 dc.$$

In a similar manner we have for the total stress produced by the sum of the components parallel to the plane of the plate,

$$P = 2\delta\Omega^2 \frac{b}{a} \int_{d-\frac{1}{2}c}^{d+\frac{1}{2}c} \int_{0}^{a} (a^2 - x^2)^{\frac{1}{2}}\, xdxdz$$

$$= \tfrac{2}{3}\delta\Omega^2 ba^2 c.$$

Hence, since the moment of inertia of the rectangular section is given by

$$I = \tfrac{1}{6}bc^3,$$

we have for the maximum intensity of stress, in absolute units,

$$\sigma = \frac{\delta\Omega^2 a^2}{3c}(c + 6d).$$

If σ be given, then obviously

$$\Omega = \sqrt{\left(\frac{3c\sigma}{\delta a^2 (c + 6d)}\right)}.$$

[*Jas. S. Miller.*]

EXERCISES.

195

UNDER what conditions will the equation

$$(a - \tfrac{1}{2})\, x^3 + \tfrac{1}{2}x^2 + x - (1 + x)\log(1 + x) = 0$$

have positive real roots? [*R. S. Woodward.*]

196

WHAT values of x and y will render the following value of u a numerical maximum: —

$$\begin{aligned} u = &+ 1.59 \cos(3x + \tfrac{3}{2}y + 67^\circ) \sin \tfrac{3}{2}y \\ &+ 1.16 \cos(6x + 3y + 329^\circ) \sin 3y \\ &+ 0.74 \cos(9x + \tfrac{9}{2}y + 323^\circ) \sin \tfrac{9}{2}y ? \end{aligned}$$

[*R. S. Woodward.*]

197

INTEGRATE
$$\frac{xd^2x}{dt^2} = a\,[b^3 - (b-t)^3],$$
a and b being constants. [*R. S. Woodward.*]

198

GIVEN
$$\tan\theta = -\frac{\theta x}{1-x}$$
Express θ in a series of ascending powers of x, and show when the series will be converging. [*R. S. Woodward.*]

199

GIVEN
$$ay = \int_{-ab}^{+ab} e^{-x^2} dx.$$
Find y when $a = 0$, without resort to series. [*R. S. Woodward.*]

200

FIND V and $\dfrac{d^2(rV)}{dr^2}$ from the equation
$$\frac{d^nV}{dr^n} = (-1)^{n+1} f(n)\, Mr^{-(n+1)} + \frac{c}{\pi}\,\frac{\sin n\pi}{n},$$
in which c and M are constants, n is zero or any positive integer, and
$$\begin{aligned} f(n) &= 1 \quad \text{for } n = 0, \\ &= n! \ \text{ for } n > 0. \end{aligned}$$
[*R. S. Woodward.*]

201

SHOW that
$$\int_0^\beta \sin x dx \int_0^x \left(\frac{\cos y - \cos\beta}{\cos y - \cos x}\right)^{\frac{1}{2}} dy$$
$$+ \int_\beta^\pi \sin x dx \int_0^\beta \left(\frac{\cos y - \cos\beta}{\cos y - \cos x}\right)^{\frac{1}{2}} dy = 2\pi \sin^2 \tfrac{1}{2}\beta.$$

Show, also, that the first of these two definite integrals is

$$2(\sin\beta - \beta\cos\beta). \qquad [R. S. Woodward.]$$

202

GIVEN $\theta\tan\theta = x$ and $x < 1$;

find the nth term of the expansion of θ in a series of ascending powers of x, and develop $2x\sqrt{(\theta^2 + x^2)}\,[\theta x + \theta(\theta^2 + x^2)]$ to terms of the fourth order in x.

[*R. S. Woodward.*]

203

IN an ellipse whose major and minor axes are AB, DN, foci F, F', centre C, and semi-latus rectum FG; BG intersects CD in O. Prove $CO = AF$.

[*P. H. Ryan.*]

204

FROM the vertices of the triangle ABC are drawn transversals AA', BB', CC', intersecting in O. Prove

$$\frac{AO}{OA'} + \frac{BO}{OB'} + \frac{CO}{OC'} > 6. \qquad [P. H. Ryan.]$$

205

A TRIANGLE PQR is inscribed in an ellipse, the two sides PQ and PR pass through the foci, and the line QR produced meets the tangent at P in the point S; show that the polar of S, with respect to a concentric circle through P, passes through the centre of curvature of the ellipse at the point P. [*R. H. Graves.*]

206

IF from any point four normals be drawn to each of the hyperbolas,

$$x^2 - y^2 = a^2 \text{ and } xy = k^2,$$

the centres of mean position of the feet of the two sets of normals are coincident.

[*R. H. Graves.*]

207

P and Q are middle points of opposite edges of a tetraedron. A plane through PQ intersects two other opposite edges in M and N. Show that MN is bisected by PQ. [*Asaph Hall.*]

208

PROVE that the surface of an oblate spheroid whose major semiaxis is a and eccentricity e, is equal to

$$2a^2\pi\left[1 + \text{Naperian log}\left(\frac{1+e}{1-e}\right)^{\frac{1-e^2}{2e}}\right].$$

[*R. S. Woodward.*]

209

SHOW whether the radius vector of a particle describing a curve which is the resultant of any two simple harmonic motions, at right angles to one another and in the same plane, sweeps out areas which are proportional to the times.

[*Jas. S. Miller.*]

210

THE length of a bar having the temperature t_0 is l_0. Prove that when the bar rises to the temperature t, the length becomes

$$l = l_0 e^{\varepsilon (t - t_0)},$$

e being the Naperian base of logarithms, and ε the linear coefficient of expansion, assumed constant. If ε is not constant, but a function of t, specify the conditions under which

$$l = l_0 e^{[a(t - t_0) + b(t - t_0)^2 + \ldots]},$$

and

$$l = l_0 [1 + A(t - t_0) + B(t - t_0)^2 + \ldots],$$

hold true, $a, b, \ldots, A, B, \ldots$ being constants. [*R. S. Woodward.*]

211

WHAT would be the height h of the earth's atmosphere, if its density at any height z were given by the formula

$$\rho = \rho_0 h^{-1} (h - z),$$

ρ_0 being the density at the earth's surface? [*R. S. Woodward.*]

212

ACCORDING to the Gaussian theory of a lens or system of lenses, we have the relation of conjugate distances

$$f_1^{-1} + f_2^{-1} = f^{-1},$$

and

$$s = s_0 f_2 f_1^{-1},$$

in which f is constant, and s and s_0 are any two corresponding linear dimensions of the image and object. Show that

$$\frac{ds}{s} = -\frac{df_1}{f_1} \cdot \frac{f_2}{f}, \quad \text{or} \quad +\frac{df_2}{f_2} \cdot \frac{f_1}{f},$$

and that

$$\frac{\Delta s}{s} = -\left(\frac{f_1 + f_2}{f_1^2}\right) \Delta f_1 + \left(\frac{f_1 + f_2}{f_1^2}\right)^2 (\Delta f_1)^2 + \ldots.$$

[*R. S. Woodward.*]